

The CKM Parameters

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Abstract

The Cabibbo-Kobayashi-Maskawa matrix is a key element to describe flavour dynamics in the Standard Model. With only four parameters, this matrix is able to describe a large range of phenomena in the quark sector, such as CP violation and rare decays. It can thus be constrained by many different processes, which have to be measured experimentally with a high accuracy and computed with a good theoretical control. With the advent of the B factories and the LHCb experiment taking data, the precision has significantly improved recently. The most relevant experimental constraints and theoretical inputs are reviewed and fits to the CKM matrix are presented for the Standard Model and for some topical model-independent studies of New Physics.

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1 Introduction

The study of elementary particles and their electromagnetic, weak and strong interactions has led to a particularly successful theory, the Standard Model (SM). It has been extensively tested, culminating with the recent discovery of the Higgs boson [1, 2] at the Large Hadron Collider (LHC). In the development of this description, quark flavour physics has played a central role in two different aspects. First, the SM embeds the Kobayashi-Maskawa mechanism: the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [3, 4] arising in charged weak interactions provides a single source of all observed differences between particles and antiparticles, i.e., CP violation in the quark sector. Second, flavour-changing currents (in particular, neutral ones) have repeatedly shown the path for new, heavier, degrees of freedom (charm quark, weak gauge bosons, top quark) before their discovery.

Yet, the SM fails with some key aspects: why is there such a large number of parameters for quark masses and CKM mixing matrix, spanning such a wide range of values ? why are the electroweak and strong interactions treated separately ? why is antimatter so absent from the observed universe while the amount of CP violation in the SM is too small to produce the observed matter-antimatter asymmetry [5–8] ? New Physics (NP) extensions of the SM are expected to address these issues by including heavier particles related to higher-energy phenomena. The related shorter-distance interactions would have immediate consequences not only in production experiments at high energies, but also through deviations from the SM predictions in flavour processes (new sources of CP violation, interferences between SM and NP contributions).

Therefore, a precision study of the CKM matrix is certainly desirable from a practitioner’s point of view: performing the metrology of the SM parameters yields accurate predictions for weak transitions, including CP -violating processes. But it is also required from a more theoretical point of view: the mixing due to the CKM matrix in weak processes has a very simple and constrained structure in the SM and is generally affected significantly by NP extensions, constituting a very potent probe of models beyond the SM. The need for an accurate determination of the CKM matrix has led to an impressive effort from the experimental community, with the extensive work performed at the BaBar and Belle experiments, the large data samples available at the LHC and the advent of the high-luminosity Belle-II B factory. The theoretical community has also made remarkable progress in its understanding of strong and weak interactions of the quarks, both analytically (in particular through the development of effective theories) and numerically (with an outstanding improvement of lattice simulations of QCD). Very high precision measurements of CKM parameters are thus both needed and accessible currently, and they are the object of this review. The theoretical grounds related to the CKM matrix are given in Sec. 2, the main experimental constraints on its parameters briefly reviewed in Sec. 3, and examples of global analyses of the CKM matrix and the impact of NP contributions presented in Sec. 4.

2 The CKM matrix

2.1 Structure of the CKM matrix

In the SM, the Lagrangian coupling of the Higgs boson to the quark fields yields (after electroweak symmetry breaking)

$$\mathcal{L}_M^q = -(M_d)_{ij} \overline{D'_{Li}} D'_{Rj} - (M_u)_{ij} \overline{U'_{Li}} U'_{Rj} \quad (1)$$

where i, j are family indices with $U' = (u', c', t')$, $D' = (d', s', b')$ and L, R indicates the components with left- and right-handed chiralities. The primes remind that these fields are not necessarily the mass eigenstates of the theory. The matrices M_u and M_d are related to the Yukawa coupling matrices as $M_q = vY^q/\sqrt{2}$, with v the vacuum expectation value of (the neutral

component of) the Higgs field. At this stage, M_u and M_d are general complex matrices to be diagonalised using singular value decomposition $M_q = V_{qL}^\dagger m_q V_{qR}$ with $V_{L,R}$ unitary and m_q diagonal, real and positive. The mass eigenstates are identified as $U_L = V_{uL} U_L'$, $U_R = V_{uR} U_R'$, and similarly for D .

Expressing the interactions of quarks with gauge bosons in terms of mass eigenstates does not modify the structure of the Lagrangian in the case of neutral gauge bosons, but it affects charged-current interactions of quarks with W^\pm , finally described by the Lagrangian

$$\mathcal{L}_{W^\pm} = -\frac{g}{\sqrt{2}} \bar{U}_i \gamma^\mu \frac{1 - \gamma^5}{2} (V_{\text{CKM}})_{ij} D_j W_\mu^\pm + h.c., \quad (2)$$

where g is the electroweak coupling constant and $V_{\text{CKM}} = V_{uL}^\dagger V_{dL}$ is the unitary CKM matrix

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (3)$$

The CKM matrix originates from the misalignment in flavour space of the up and down components of the $SU(2)_L$ quark doublets of the SM (as there is no dynamical mechanism in the SM to enforce $V_{uL} = V_{dL}$). The $V_{\text{CKM},ij}$ CKM matrix elements (hereafter V_{ij}) represent the couplings between up-type quarks $U_i = (u, c, t)$ and down-type quarks $D_j = (d, s, b)$. There is some arbitrariness in conventions to define this matrix. In particular, the relative phases among the left-handed quark fields can be redefined, reducing the number of real parameters describing this unitary matrix from 3 moduli and 6 phases down to 3 moduli and 1 phase. Since CP conjugate processes correspond to interaction terms in the Lagrangian related by Hermitian conjugation, the presence of a phase, and thus the complex nature of the CKM matrix, may induce differences between rates of CP conjugate processes, leading to CP violation.

According to experimental evidence, transitions within the same generation are characterised by V_{CKM} elements of $\mathcal{O}(1)$. Those between the first and second generations are suppressed by a factor $\mathcal{O}(10^{-1})$; those between the second and third generations by a factor $\mathcal{O}(10^{-2})$; and those between the first and third generations by a factor $\mathcal{O}(10^{-3})$. This hierarchy can be expressed by defining the four phase-convention-independent quantities

$$\lambda^2 = \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \quad A^2 \lambda^4 = \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \quad \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}. \quad (4)$$

An alternative convention exists in the literature for the last two CKM parameters, corresponding to

$$\rho + i\eta = \frac{V_{ub}^*}{V_{us}V_{cb}^*} = \left(1 + \frac{1}{2}\lambda^2\right) (\bar{\rho} + i\bar{\eta}) + \mathcal{O}(\lambda^4) \quad (5)$$

The CKM matrix can be expanded in powers of the small parameter λ (which corresponds to $\sin \theta_C \simeq 0.22$) [9] exploiting the unitarity of V_{CKM} to highlight its hierarchical structure. This expansion yields the following expansion up to $\mathcal{O}(\lambda^6)$

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\bar{\rho} + i\bar{\eta})] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (\bar{\rho} + i\bar{\eta})] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\bar{\rho} + i\bar{\eta})] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}. \quad (6)$$

The CKM matrix is complex, and hence CP violation is allowed, if and only if $\bar{\eta}$ differs from zero. To lowest order the Jarlskog parameter measuring CP violation in a convention-independent manner [10]

$$J_{CP} \equiv |\Im(V_{i\alpha}V_{j\beta}V_{i\beta}^*V_{j\alpha}^*)| = \lambda^6 A^2 \bar{\eta}, \quad (i \neq j, \alpha \neq \beta), \quad (7)$$

is directly related to the CP -violating parameter $\bar{\eta}$, as expected.

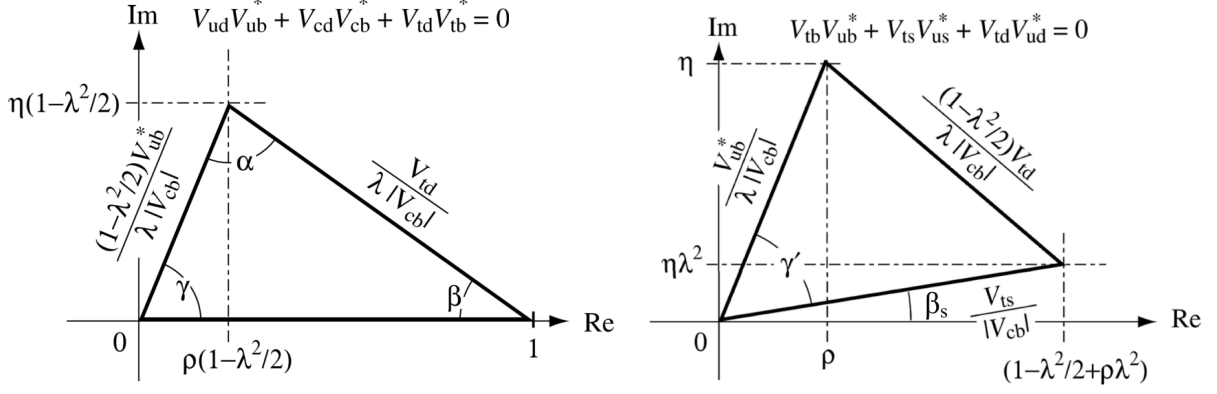


Figure 1: Representation in the complex plane of the non-squashed triangles obtained from the off-diagonal unitarity relations of the CKM matrix Eq. (8). On the left, the three sides are rescaled by $V_{cd}V_{cb}^*$, on the right, by $V_{us}V_{cb}^*$.

2.2 The unitarity triangle

To represent the knowledge on the four CKM parameters, it proves useful to exploit the unitarity condition of the CKM matrix, $V_{\text{CKM}}^\dagger V_{\text{CKM}} = V_{\text{CKM}} V_{\text{CKM}}^\dagger = \mathbb{I}$. This corresponds to a set of 12 equations: 6 for diagonal terms and 6 for off-diagonal terms. In particular, the equations for the off-diagonal terms can be represented as triangles in the complex plane, all characterised by the same area $J_{CP}/2$. Only two out of these six triangles have sides of the same order of magnitude, $\mathcal{O}(\lambda^3)$, i.e., are not squashed

$$\underbrace{V_{ud}V_{ub}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}V_{cb}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}V_{tb}^*}_{\mathcal{O}(\lambda^3)} = 0, \quad \underbrace{V_{ud}V_{td}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{us}V_{ts}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{ub}V_{tb}^*}_{\mathcal{O}(\lambda^3)} = 0. \quad (8)$$

These two triangles in the complex plane are represented in Fig. 1. In particular, the triangle defined by the former equation and rescaled by a factor $V_{cd}V_{cb}^*$ is commonly referred to as *the* unitarity triangle (UT). The sides of the UT are given by

$$R_u \equiv \left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}, \quad R_t \equiv \left| \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right| = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2}. \quad (9)$$

The parameters $\bar{\rho}$ and $\bar{\eta}$ are the coordinates in the complex plane of the non-trivial apex of the UT, the others being (0, 0) and (1, 0). CP -violation in the quark sector ($\bar{\eta} \neq 0$) is translated into a non-flat UT. The angles of the UT are related to the CKM matrix elements as

$$\alpha \equiv \phi_2 \equiv \arg \left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right) = \arg \left(-\frac{1 - \bar{\rho} - i\bar{\eta}}{\bar{\rho} + i\bar{\eta}} \right), \quad (10)$$

$$\beta \equiv \phi_1 \equiv \arg \left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right) = \arg \left(\frac{1}{1 - \bar{\rho} - i\bar{\eta}} \right), \quad (11)$$

$$\gamma \equiv \phi_3 \equiv \arg \left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right) = \arg (\bar{\rho} + i\bar{\eta}). \quad (12)$$

The above equations show the two coexisting notations in the literature. Because it involves the CKM matrix $V_{Ud}V_{Ub}^*$ (with $U = u, c, t$), the UT arises naturally in discussions of B^0 -meson transitions.

The second non-squashed triangle has similar characteristics with respect to the UT, but it involves $V_{uD}V_{tD}^*$ (with $D = d, s, b$) and is not immediately associated to a neutral meson. A

modified triangle, shown in Fig. 1, can be defined, where all sides are rescaled by $V_{us}V_{cb}^*$. Up to $O(\lambda^4)$ corrections, its apex is located the point (ρ, η) and it is tilted with respect to the horizontal axis by an angle

$$\beta_s \equiv \arg \left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right) = \lambda^2 \bar{\eta} + \mathcal{O}(\lambda^4). \quad (13)$$

As mentioned before, neutral mesons with other flavour content (B_s^0, D^0, K^0) would correspond to other, squashed, triangles, with the same area and with some of their angles related to those defined above. For instance, β_s occurs naturally in the B_s^0 unitarity triangle defined from $V_{Us}V_{Ub}^*$ (with $U = u, c, t$). All these representations are particular two-dimensional projections of the four parameters describing the CKM matrix, which can be constrained combining experimental and theoretical information.

3 Individual constraints

3.1 Types of constraints

Due to its economical structure in terms of only 4 parameters and its consequences for CP violation, the CKM matrix can be determined through many different quark transitions. They correspond to $\Delta F = 1$ decays or $\Delta F = 2$ processes related to neutral-meson mixing.

Extensive measurements have been performed on K , D , and B mesons at different experiments. Constraints coming from K mesons or unflavoured particles are mostly obtained from dedicated experiments, among which NA48 [11], KLOE [12, 13] and KTeV feature prominently. Measurements of CKM parameters from D and B mesons were pioneered by ARGUS [14] at DESY, CLEO and CLEO-c [15] at Cornell, followed by the so-called B factories BABAR [16] at SLAC and Belle [17] at KEK. They operated mostly a centre-of-mass energy corresponding to the mass of the $\Upsilon(4S)$ resonance. Significant contributions also came from the CDF and DØ experiments at FNAL [18], especially those involving B_s^0 mesons which are not accessible at the $\Upsilon(4S)$ resonance. These experiments are now terminated, while Belle is being upgraded [19]. Physics with b and c hadrons is now dominated by the LHCb experiment [20] at the LHC. The general-purpose detector experiments ATLAS [21] and CMS [22] contribute in selected areas, while the BESIII experiment [23] also provides many results for charm hadrons.

Experimental measurements are related to an amplitude that sums several terms, each containing CKM factors multiplied by quantities describing the quark transition and the hadronisation of quarks into observable mesons or baryons. Whether a given process is relevant depends on the experimental and theoretical accuracy that can be reached. Due to the complexity of long-distance strong interactions, it proves easier to select processes with a limited number of hadrons in the initial or final state, or to select observables (typically ratios) out of which long-distance QCD uncertainties cancel.

In the first case, (exclusive) CP -conserving processes with at most one hadron in the initial and the final state are considered. After integrating out heavy degrees of freedom (in particular weak gauge bosons) using the effective Hamiltonian formalism [24], the long-distance hadronic contribution can be parametrised in terms of relatively simple quantities which are accessible through theoretical tools (lattice QCD simulations, effective field theories): decay constants for leptonic decays, form factors for semileptonic decays, bag parameters (matrix elements of four-quark effective operators between a meson and its anti-meson) for neutral-meson mixing. It proves often useful to consider ratios of observables related by $SU(3)$ flavour symmetry, as many experimental and theoretical uncertainties drop in such ratios. For a few (inclusive) processes, a sum over all possible final states is performed: quark-hadron duality can then be invoked to compute the effects of the strong interaction perturbatively. For this first type of observables,

Table 1: A partial list of measurements generally used to determine the CKM parameters, the combination of CKM parameters constrained, and the theoretical inputs needed. They are classified according to the dominant type of uncertainties (experimental or theoretical) and the type of processes involved (tree or loop).

	Dominated by experimental uncertainties		Dominated by theoretical uncertainties	
	Process	Constraint	Process	Constraint
Tree	$B \rightarrow D^{(*)} K^{(*)}$	γ	$B \rightarrow D^{(*)} \ell \nu$	$ V_{cb} $ vs form factor $F^{B \rightarrow D^{(*)}}$
			$B \rightarrow X_c \ell \nu$	$ V_{cb} $ vs OPE
			$B \rightarrow \pi \ell \nu$	$ V_{ub} $ vs form factor $F^{B \rightarrow \pi}$
			$B \rightarrow X_u \ell \nu$	$ V_{ub} $ vs (OPE)
			$M \rightarrow \ell \nu$	$ V_{UD} $ vs decay constant f_M
			$M \rightarrow N \ell \nu$	$ V_{UD} $ vs form factor $F^{M \rightarrow N}$ or $M \rightarrow N$ amplitude
Loop	$B \rightarrow (c\bar{c}) K^{(*)}$	β	ϵ_K ($K\bar{K}$ mix)	$V_{ts}V_{td}^*$ and $V_{cs}V_{cd}^*$ vs bag parameter B_K
	$B \rightarrow \pi\pi, \rho\pi, \rho\rho$	α	Δm_d ($B^0\bar{B}^0$ mix)	$ V_{tb}V_{td}^* $ vs bag parameter B_{B^0}
	$B_s^0 \rightarrow J/\psi\phi$	β_s	Δm_s ($B_s^0\bar{B}_s^0$ mix)	$ V_{tb}V_{ts}^* $ vs bag parameter $B_{B_s^0}$

where significant hadronic uncertainties must be assessed carefully, the resulting constraints are generally set on the modulus of a given CKM matrix element, and is dominated by theoretical uncertainties.

In the second case, CP -violating quantities are devised by comparing a process and its CP -conjugate. Since the strong interaction conserves CP , the same hadronic amplitudes are involved and may cancel in well-designed observables such as CP asymmetries, measuring either CP violation in hadron decays, in neutral meson mixing or in the interference between these two types of processes. This second type of observables, from which most of the hadronic uncertainties are absent, often yields information about one particular angle of the UT, dominated by experimental uncertainties. Large CP asymmetries are associated with the non-squashed UT and thus occur mainly for B -meson processes (often with small branching ratios due to CKM-suppressing factors).

The processes for which a good accuracy can be reached both experimentally and theoretically are summarised in Tab. 1. They are used to assess the validity of the Kobayashi-Maskawa mechanism for CP violation and to perform the metrology of the CKM parameters, assuming the validity of the SM. It is to be noted that $\Delta F = 2$ meson-mixing corresponds to a flavour-changing neutral current, and as such, it is forbidden at tree level and is only mediated by loop processes in the SM. On the other hand, $\Delta F = 1$ decays can be either related to tree processes (typically, leptonic and semileptonic decays) or also involve loop processes (such as hadronic decays).

The potential sensitivity to physics beyond the SM is not the same for all processes. When discussing potential NP effects, it often proves interesting to perform the metrology of the CKM matrix using only tree-level processes, and to exploit loop processes in order to constrain additional NP effects. One may also consider additional ultrarare decays and processes that are not experimentally measured with a sufficient accuracy to constrain the CKM matrix in the SM, but are very sensitive to NP, for instance the rare $B_s^0 \rightarrow \mu\mu$ and $K \rightarrow \pi\nu\nu$ decays or the B_s^0 width difference $\Delta\Gamma_s$. This issue will be discussed further in Sec. 4.2.

3.2 Moduli from leptonic and semileptonic decays $\Delta F = 1$

The following moduli can be determined accurately from (CP -averaged) branching ratios of exclusive leptonic and semileptonic decays.

3.2.1 Transitions among the first and second generations

The CKM matrix element $|V_{us}|$ is efficiently constrained by $K^- \rightarrow \ell^- \bar{\nu}$, $K \rightarrow \pi \ell \bar{\nu}$ and $\tau \rightarrow K^0 \nu_\tau$ decays [25]. Decay constants and form factors are known from lattice QCD simulations [26], while radiative corrections have been determined with a high accuracy based on Chiral Perturbation Theory [27].

The matrix elements $|V_{cd}|$ and $|V_{cs}|$ are constrained by D , D^+ and D_s^+ leptonic and semileptonic decays. The precision of the leptonic decays [28–31] (where the lepton is often a muon, but can be a τ lepton in the case of the D_s^+ meson [31–33]) is dominated by experimental uncertainties. Conversely, the semileptonic $D \rightarrow K \ell \nu$ and $D \rightarrow \pi \ell \nu$ decays [34–38] have not been investigated by many lattice QCD collaborations, and their systematic uncertainties are expected to be improved to yield relevant constraints for the CKM parameters [26]. Moreover, radiative corrections have still to be investigated in detail for these processes [39, 40].

In principle, $|V_{ud}|$ could be determined by many processes, as $\pi^+ \rightarrow e^+ \nu$, $\pi^+ \rightarrow \pi^0 e^+ \nu$, $n \rightarrow p e^- \bar{\nu}$. Yet they exhibit a poor experimental accuracy for our purposes (pion leptonic or semileptonic decays) or their measurements in different experimental settings are not compatible and cannot be averaged meaningfully (neutron lifetime) [25]. It turns out that the most accurate determination comes from nuclear super-allowed $0^+ \rightarrow 0^+$ β decays [41]. The current determination is based on a large set of nuclei and relies on sophisticated estimates of different corrections (electroweak radiative, nuclear structure, isospin violation) from dedicated nuclear physics approaches.

3.2.2 $|V_{ub}|$ and $|V_{cb}|$

The determination of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ provides important closure tests of the UT. It is best performed in semileptonic $b \rightarrow (u, c) \ell \nu$ decays ($\ell = e, \mu$), where there are no hadronic uncertainties related to the decay of the emitted W boson. Unfortunately, a well-known discrepancy is present between the determinations obtained from exclusive decays and inclusive modes [42], which are treated with different tools. In the case of V_{cb} , there is no complete lattice QCD determination of the $B \rightarrow D^{(*)} \ell \nu$ form factors which are required to analyse the corresponding experimental exclusive measurements [42–45]. Heavy Quark Effective Theory (HQET) is required, expanding the form factors in powers of $1/m_b$ and $1/m_c$ in order to simplify their expression and constrain their dependence on the lepton energy, complemented with lattice QCD estimates of some of the HQET parameters. For the inclusive decay $B \rightarrow X_c \ell \nu$ [46–49], Operator Product Expansion allows expressing the decay rate as a series in $1/m_b$ and $1/m_c$ [50], with matrix elements that can be fitted from leptonic and hadronic moments of the branching ratio [51].

In the case of $|V_{ub}|$, the exclusive determination benefits from lattice QCD computations for the vector $B \rightarrow \pi \ell \nu$ form factor [52–54], which can be combined with measurements of the differential decay rate [42, 55–57] in order to determine V_{ub} . The inclusive determination [58–62] is more challenging. The full decay rate cannot be accessed since a cut in the lepton energy must be performed to eliminate the huge $b \rightarrow c \ell \nu$ background. The Operator Product Expansion must be modified, introducing poorly known shape functions describing the b -quark dynamics in the B meson [63–67]. They can be constrained partly from $B \rightarrow X_s \gamma$ and raise questions concerning the convergence rate of the series in $1/m_b$ [68].

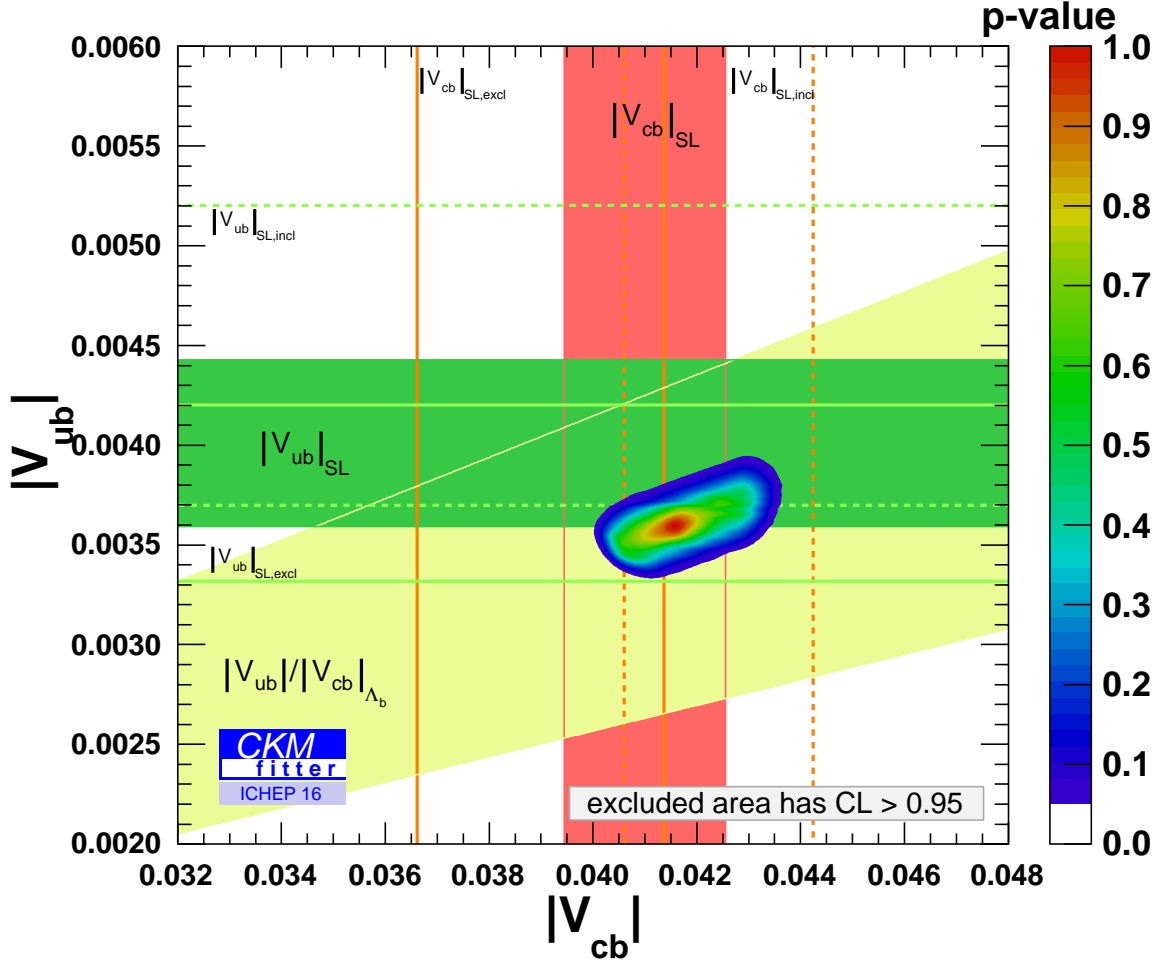


Figure 2: Experimental situation for $|V_{ub}|$ and $|V_{cb}|$. The experimental measurements from exclusive (inclusive) measurements are represented by bands with solid (dashed) lines, and their average is represented by the coloured bands. The yellow diagonal band corresponds to the constraint from Λ_b^0 decays. The oval region indicates the 95% CL region is the indirect determination of $|V_{ub}|$ and $|V_{cb}|$ from a global fit including none of these measurements. Figure from Ref. [76].

These determinations leads to a long-standing discrepancy between inclusive and exclusive determinations for $|V_{ub}|$ and $|V_{cb}|$. The current situation is that global fits (discussed in Sec. 4) use averages of both kinds of determination as inputs, but their outcome favours exclusive measurements for $|V_{ub}|$ and inclusive measurements for $|V_{cb}|$, as can be seen in Fig. 2.

Additional decay modes need to be added to get a global picture for $|V_{ub}|$ and $|V_{cb}|$. The leptonic decay $B \rightarrow \tau \nu_\tau$ has been studied at B factories [69–72], favouring values closer to the inclusive determination. The value of this branching ratio used to be at odds with expectations from global fits [73], but recent determinations from Belle reduced the discrepancy down to 1.2σ . In addition, the LHCb collaboration has recently used Λ_b^0 baryon decays for the first time [74]. The decay rates of $\Lambda_b^0 \rightarrow p \mu^- \nu$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \nu$ are compared to determine the ratio $|V_{ub}/V_{cb}|$, using the available lattice QCD estimates of the six different form factors involved [75]. The overall situation is shown in Fig. 2, where the constraints from inclusive and exclusive determinations of $|V_{ub}|$, $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ are shown.

3.3 Unitarity Triangle angles from CP -violating measurements

The UT angles described in the following subsections can be determined experimentally from CP -violating measurements with almost no theoretical uncertainties.

3.3.1 The angle $\beta \equiv \varphi_1$

The mode which allowed for the first observation of CP violation in B decays is $B^0 \rightarrow J/\psi K_S^0$ [77, 78]. It gives access to φ_d [79], the relative phase between the decay of the B^0 meson to $J/\psi K_S^0$ and that of the oscillation of B^0 to its antiparticle \bar{B}^0 followed by the decay $\bar{B}^0 \rightarrow J/\psi K_S^0$. The measurement requires studying how the decay depends on the time between the initial production of B^0 and its decay, leaving time for evolution and potential mixing between B^0 and \bar{B}^0 mesons. In the SM, the decay is dominated by a single CKM phase, up to Cabibbo-suppressed penguin contributions, whereas the B^0 mixing is completely dominated by top-top box diagrams. Considering these two phases, the measurement of the time-dependence of this process yields $\sin 2\beta$ [80]. The B factories were optimised for its measurement [81–83] and determined [42] $\sin 2\beta^{\text{B-fact}} = 0.682 \pm 0.019$, which is the most precise constraint on the unitarity triangle (Fig. 6). Recently LHCb joined the effort, publishing their first measurement of the time-dependent CP asymmetry in the decay $B^0 \rightarrow J/\psi K_S^0$ [84] with an uncertainty competitive with the individual measurements from the B factories. The degeneracies among the values of β are lifted thanks to the $B^0 \rightarrow J/\psi K^{*0}$ mode [85, 86], where the interferences between the difference partial waves are sensitive to $\cos 2\beta$.

The measured value for $\sin 2\beta$ is slightly lower than the expectation from all other constraints on the UT [76, 87] $\sin 2\beta^{\text{indirect}} = 0.740^{+0.020}_{-0.025}$, which could be due to the so far neglected contribution from penguin topologies in the decay $B^0 \rightarrow J/\psi K_S^0$ or in other $b \rightarrow c\bar{c}s$ decays to CP eigenstates. There have been several theoretical attempts to estimate this contribution. A first possibility consists in using $SU(3)$ symmetry and assessing the size of penguin contributions from $B^0 \rightarrow J/\psi \pi^0$, $B^0 \rightarrow J/\psi \rho^0$ and $B_s^0 \rightarrow J/\psi K_S^0$ decays [88, 89], unfortunately with a limited accuracy due to the experimental input [90–93]. A fit to $B \rightarrow J/\psi P$ (with P a light pseudo-scalar meson) including $SU(3)$ breaking corrections suggests on the other hand a small contamination from penguin contributions [94]. Direct computations based on Soft-Collinear Effective Theory arguments [95] reach a similar conclusion. The final average of all charmonium data yields the very accurate value $\sin 2\beta^{\text{meas}} = 0.691 \pm 0.017$ [42].

The value of $\sin 2\beta$ can also be determined in $b \rightarrow q\bar{q}s$ transitions, with $q = d, s$, as $B^0 \rightarrow \eta' K^0$ [96, 97]. These are not allowed at tree level and thus probe the CKM mechanism in loop-induced processes, although the contamination from penguins with other CKM phases is difficult to assess in these modes [98]. The naive average of all measurements results in $\sin 2\beta^{q\bar{q}s} = 0.655 \pm 0.032$ [42], which is consistent with expectations.

A key ingredient for this and other time-dependent measurements is the ability to identify the flavour of the B meson, before it starts its evolution and mixes with its antiparticle. While at B factories the so-called flavour tagging had a high efficiency [81], the complicated hadronic environment at the LHC makes this task very challenging. The tagging performance at LHCb has continuously improved over the years thanks to a better understanding of the underlying event and to the use of modern machine-learning techniques [99–102]. These improvements combined with data from the upcoming LHC Run 2 will allow for further reduced uncertainties.

3.3.2 The angle $\alpha \equiv \varphi_2$

A precise determination of the UT angle α is challenging both at the theoretical and experimental levels. It requires the time-dependent study of $b \rightarrow u$ transitions as in $B \rightarrow \pi\pi$, $B \rightarrow \rho\pi$ or $B \rightarrow \rho\rho$, which are affected by $b \rightarrow d$ or $b \rightarrow s$ penguin topologies, depending on the final state

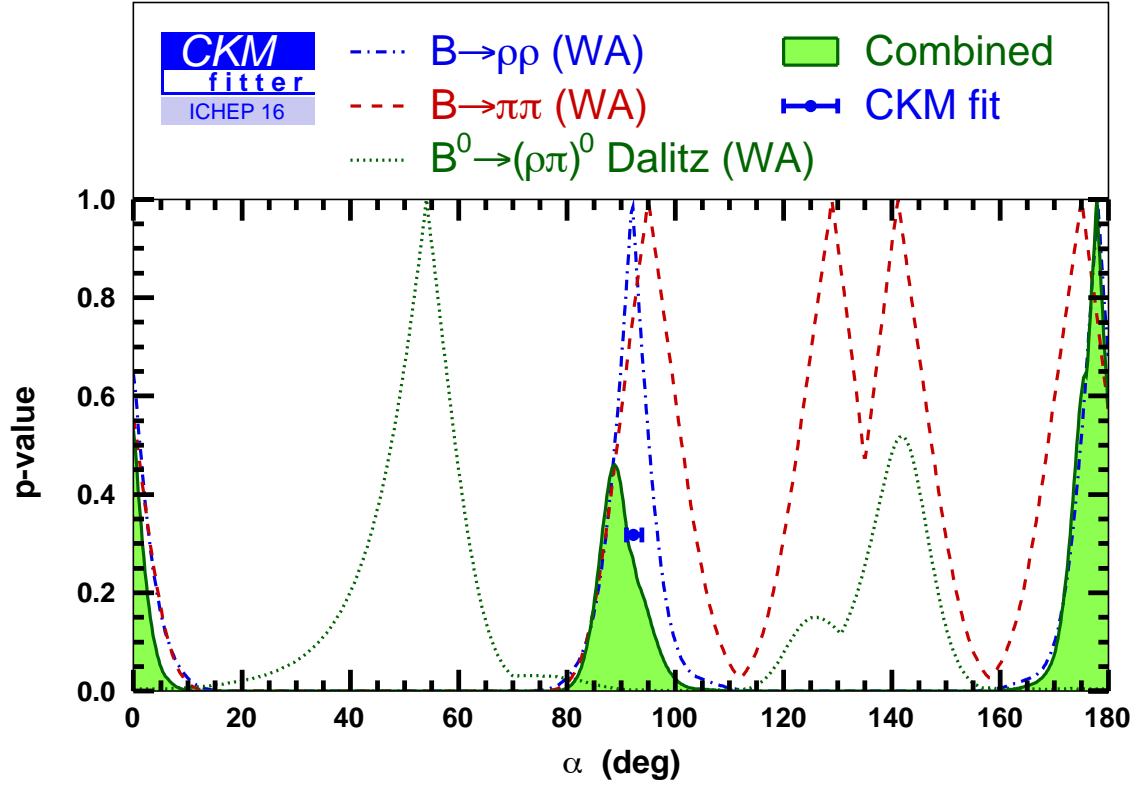


Figure 3: Constraints on the CKM angle α from $B \rightarrow \pi\pi$, $B \rightarrow \rho\pi$ and $B \rightarrow \rho\rho$. The combination of the constraints and the outcome of the global fit are also represented. Fig. from Ref. [76]

considered. The interference between B^0 - \bar{B}^0 mixing and decay amplitudes would provide a measurement of $\pi - \beta - \gamma = \alpha$ (using unitarity) in the absence of penguin contributions. In practice, this penguin pollution is present and must be constrained by determining the magnitude and relative phase of hadronic amplitudes before determining the angle α , with the help of isospin symmetry [103, 104]. For $B \rightarrow \pi\pi$ [105–111], all three possible channels are considered, and isospin symmetry can be used to relate the hadronic amplitudes, leading to triangular relations. From the measurements of branching ratios and CP asymmetries, two triangles can be reconstructed for B^+ , B^0 decays and B^- , \bar{B}^0 , respectively, with a relative angle corresponding to α , up to discrete ambiguities. For the decays $B \rightarrow \rho\rho$ [112–118], a similar construction can be invoked for the (dominant) longitudinal polarisation, with the interesting feature that the penguin contamination turns out to be less important than for $\pi\pi$ modes. The decays $B \rightarrow \rho\pi$ [117–121] require a more elaborate analysis: isospin symmetry yields pentagonal relations, whereas the time-dependent $B \rightarrow \pi\pi\pi$ Dalitz plot analysis provides a large set of observables, corresponding to the parametrisation of the amplitude together with an isobar model involving the ρ line-shape. So far, a Dalitz plot analysis has been reported only for the decay mode $B^+ \rightarrow \pi^+\pi^-\pi^+$ [122]. The present average of these constraints gives $\alpha^{\text{meas}} = (88.8^{+2.3}_{-2.3})^\circ$ [76].

The different constraints are represented in Fig. 3, showing the discrete symmetries present in the $\pi\pi$ and $\rho\rho$ cases, as well as the fact that two solutions are allowed by the combination of the measurements. In addition to the statistical uncertainties of the measurements, the accuracy is limited by two main hypotheses: $\Delta I = 3/2$ contributions coming from electroweak penguins are neglected and isospin symmetry in strong interactions is not perfect [123, 124].

3.3.3 The angle $\gamma \equiv \varphi_3$

The angle γ can be obtained from tree-dominated $B \rightarrow DK$ decays, where the CP -violating phase appears in the interference of $b \rightarrow c$ (colour allowed) and $b \rightarrow u$ (colour suppressed) topologies, followed by carefully chosen D decay processes. It is the least precisely known angle of the unitarity triangle, and its determination from tree decays is considered as free from contributions beyond the SM and unaffected by hadronic uncertainties, contrary to α and β [125]. Yet its precise determination is important to test the consistency of the CKM paradigm, and to allow comparisons with determinations from modes dominated by penguin topologies.

Three different methods have been devised in order to obtain information on γ , depending on the subsequent decays of $D^{(*)}$ mesons, with a different sensitivity on the ratio of colour favoured and suppressed amplitudes. The GLW [126, 127] method considers the decay of the D meson into CP eigenstates, eliminating further hadronic uncertainties concerning the D decays. The ADS method [128, 129] considers decays of the $D^{(*)}$ meson with a pattern of Cabibbo dominance/suppression that counteracts the colour suppression/dominance of the B decay, for instance $D \rightarrow K^\mp \pi^\pm$. Finally, the GGSZ method [130] performs a Dalitz analysis of three-body $D^{(*)}$ decays, inducing a dependence on the amplitude model for $D^{(*)}$ decays.

For the last two methods, additional information on the strong phase structure in multi-body D decays is required, which is provided by CLEO-c [131–133]. LHCb have performed several measurement using the GLW/ADS [134–138] and GGSZ [139, 140] methods with various B^0 and B^+ decays, as well as a time-dependent $B_s^0 \rightarrow D_s^\pm K^\mp$ analysis [141, 142]. As some systematic uncertainties are correlated among analyses, LHCb have performed a combination yielding $\gamma = (72.2^{+6.8}_{-7.3})^\circ$ [143].

Similarly, the B factories BaBar and Belle have performed combinations of their measurements [144, 145] and obtain $\gamma = (67 \pm 11)^\circ$ [81]. The combination of the values for γ yields $\gamma^{\text{meas}} = (72.1^{+5.4}_{-5.8})^\circ$, with the confidence level curves shown in Fig. 4. As there is no irreducible theoretical uncertainty on the determination of γ [125], there is a large room for more precision measurements of this quantity.

3.3.4 The angle φ_s

By analogy with the measurement of $\sin 2\beta$ related to B^0 mixing, a CP -violating phase φ_s related to B_s^0 mixing can be determined through time-dependent measurements of $b \rightarrow c\bar{s}s$ decays. This phase is equal to $-2\beta_s \equiv -2 \arg[-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*] = -0.0370^{+0.0006}_{-0.0007}$ rad in the SM [76], neglecting sub-leading penguin contributions. It has been measured using the flavour eigenstate decay $B_s^0 \rightarrow J/\psi \phi$ with $J/\psi \rightarrow \mu^+\mu^-$ and $\phi \rightarrow K^+K^-$ by CDF [146], DØ [147], CMS [148] and ATLAS [149]. LHCb uses the decay $B_s^0 \rightarrow J/\psi K^+K^-$ (including $B_s^0 \rightarrow J/\psi \phi$) in a polarisation-dependent way [150] and the pure CP -odd decay $B_s^0 \rightarrow J/\psi \pi^+\pi^-$ [151, 152]. The current constraints on φ_s and the decay width difference $\Delta\Gamma_s = \Gamma_L - \Gamma_H$ are shown in Fig. 5.

Similarly to the β case, the SM prediction $\varphi_s^{c\bar{c}s} = -2\beta_s$ assumes tree-dominated decays. With the increasing precision on the CKM parameters, the effects of suppressed penguin topologies cannot be neglected any more [89, 156–160]. Cabibbo-suppressed decay modes, where these topologies are relatively more prominent can be used to constrain such effects. Ways of using selected measurements enabling the sizes of penguin amplitudes to be constrained have been described in Refs. [88, 94, 95, 161, 162]. The LHCb collaboration is pursuing this programme, with studies of the decays $B_s^0 \rightarrow J/\psi K_s^0$ [163] and $B_s^0 \rightarrow J/\psi \bar{K}^{*0}$ [164].

Another interesting test of the SM is provided by the measurement of the mixing phase $\varphi_s^{s\bar{s}s}$ with a penguin-dominated mode as $B_s^0 \rightarrow \phi\phi$. In this case the measured value is $-0.17 \pm 0.15 \pm 0.03$ rad [165], which is compatible with the SM expectation.

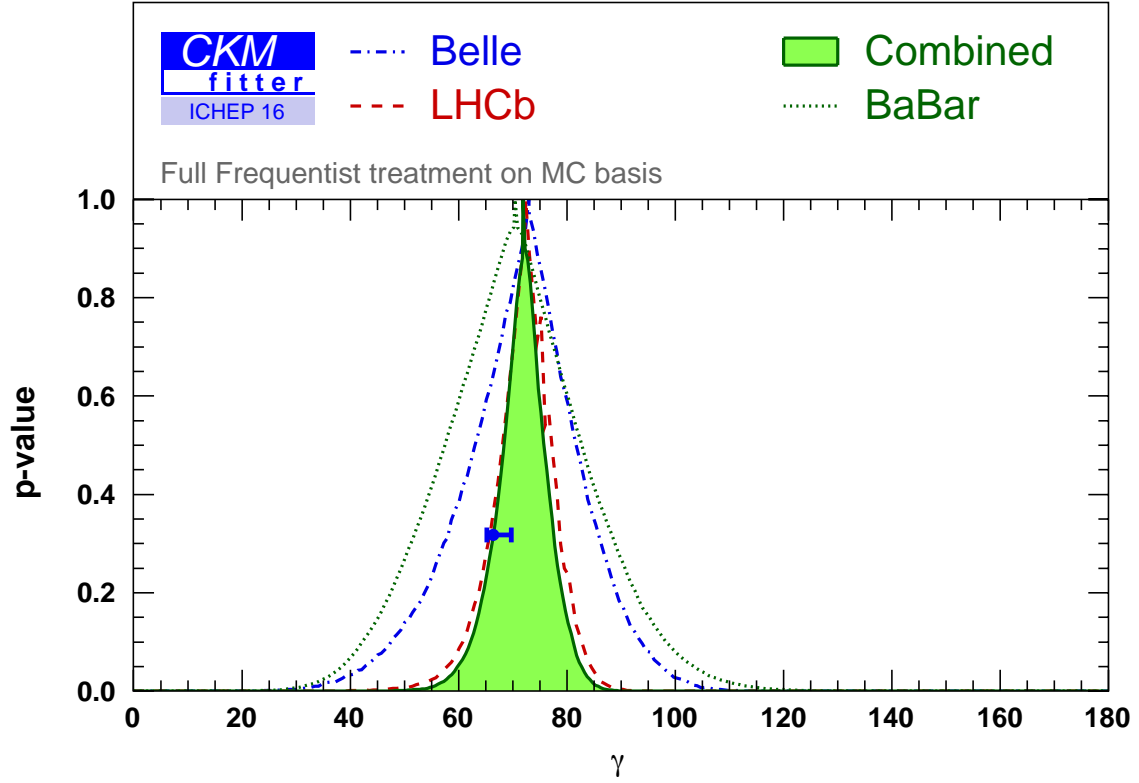


Figure 4: Constraints on the CKM angle γ from BaBar, Belle and LHCb experiments. Figure from Ref. [76]

3.4 Information from $\Delta F = 2$ transitions

$\Delta F = 2$ transitions are particularly useful both in the SM and in the search for NP, as these are flavour-changing neutral currents arising only as loops in the SM. Among the four neutral mesons available, the K^0 , B^0 and B_s^0 systems are useful for the metrology of the SM. Indeed, the mixing of the charm meson D^0 is notoriously difficult to estimate theoretically, as the GIM suppression means that it is dominated by the first two generations, and thus by long-distance QCD dynamics [166].

3.4.1 B^0 and B_s^0 systems

Neutral-meson mixing means that the flavour eigenstates P^0 and \bar{P}^0 mix into the mass eigenstates P_L and P_H denoting respectively the light and heavy mesons. This language is used to describe several observables for the B^0 and B_s^0 systems: the mass difference $\Delta m = M_H - M_L$, the width difference $\Delta\Gamma = \Gamma_L - \Gamma_H$ and the semileptonic asymmetry $a_{\text{sl}}^{d,s}$ measuring CP violation in mixing by comparing semileptonic decays of P^0 or \bar{P}^0 into “wrong-sign” leptons ((such processes can occur only if P_0 or \bar{P}_0 mixes into its antiparticle). Due to the pattern of CKM factors (suppressing charm contributions), Δm is dominated by the dispersive part of top boxes. It can be analysed within an effective Hamiltonian analysis integrating out heavy (W, Z, t, H) degrees of freedom: it amounts to a local contribution that requires the input of a single bag parameter once short-distance QCD corrections (gluon exchanges) have been taken into account [24, 167]. This explains why the mass difference Δm has been used for a long time to constrain the CKM

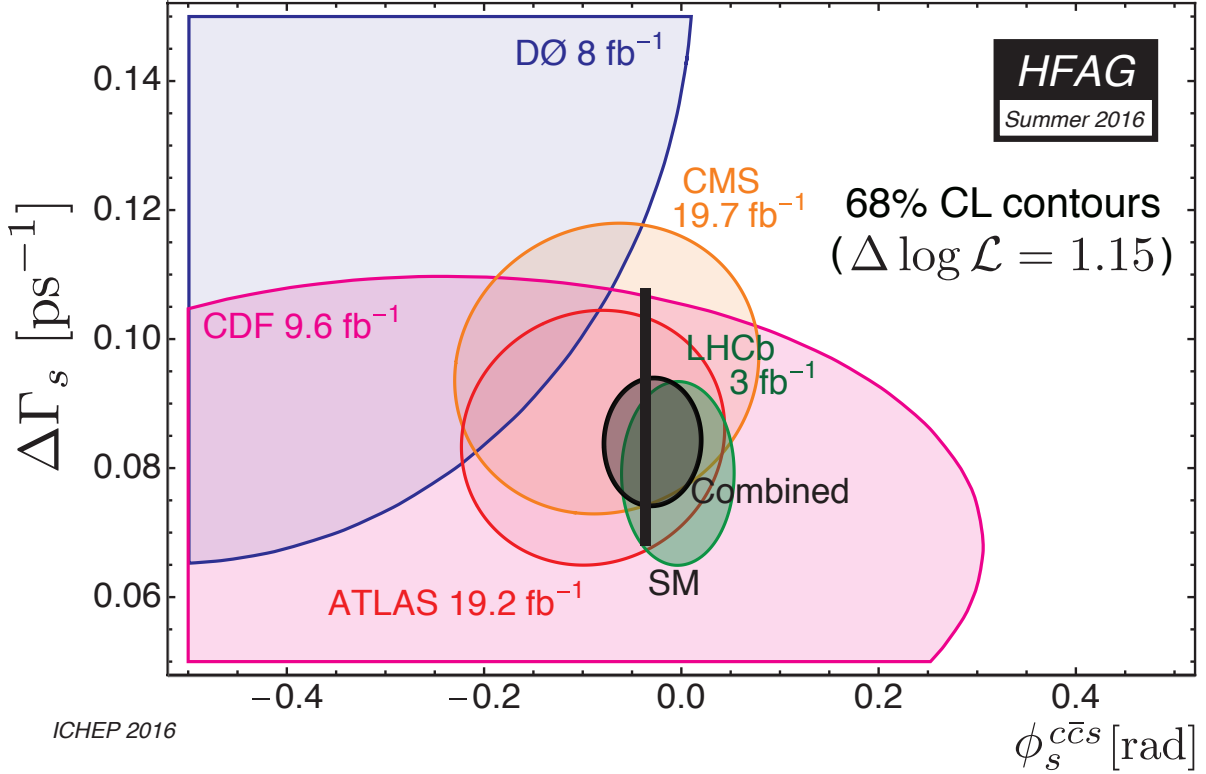


Figure 5: Constraints on $\Delta\Gamma_s$ and $\varphi_s^{c\bar{c}s}$ from various decays and experiments. Figure courtesy of HFAG [42]. The SM predictions are from Refs. [153–155].

parameters. On the contrary, $\Delta\Gamma$, related to the imaginary part of the amplitude, involves only real intermediate states: it is thus dominated by the absorptive part of charm boxes, i.e., the decays of P^0 and \bar{P}^0 into common final states. The evaluation of this non-local contribution requires a further $1/m_b$ expansion, with larger uncertainties and two hadronic bag parameters, making $\Delta\Gamma$ (and $a_{\text{sl}}^{d,s}$) harder to control theoretically [73, 154, 168–170].

The frequency of B^0 and B_s^0 mixing probes $|V_{tb}V_{tq}^*|$, with $q = d, s$, respectively. They are measured as $\Delta m_d^{\text{meas}} = 506.4 \pm 1.9 \text{ ns}^{-1}$ and $\Delta m_s^{\text{meas}} = 17.757 \pm 0.021 \text{ ps}^{-1}$ [42], which sets strong constraints on the Unitarity Triangle (UT). The accuracy is limited mainly by the determination of the corresponding bag parameters. It is more useful to consider the ratio $\Delta m_d/\Delta m_s$ which involves an $SU(3)$ breaking ratio of bag parameters, known more accurately than individual quantities from lattice simulations [26].

The B_s^0 meson system has many features in common with that of the K^0 meson, with a heavy long-lived and a light short-lived eigenstate. The a priori unknown admixture of the two states contributing to a given non-flavour-specific decay causes uncertainties in the measurement of branching fractions, for instance for the decay $B_s^0 \rightarrow \mu^+\mu^-$ [171–175]. A precise determination of the decay width difference is thus also important for the study of rare decays and constrains models of NP in $\Delta F = 2$ transitions efficiently [73, 154, 169, 176, 177].

While Δm_d and Δm_s are measured to be consistent with expectations, the DØ experiment reported an unexpectedly large dimuon asymmetry [178], which differs from the SM expectation by 3σ . This measurement is generally interpreted as a combination of the semileptonic asymmetries a_{sl}^d and a_{sl}^s in B^0 and B_s^0 decays, respectively, that measure CP violation in mixing. Direct measurements of a_{sl}^d and a_{sl}^s at B factories [179–181], DØ [182, 183] and LHCb [184, 185] are consistent with the SM prediction and in tension with the DØ asymmetry. The origin of this discrepancy is still under investigation [186], as will be recalled in Sec. 4.2.1.

3.4.2 The K^0 system

The pattern of CKM factors requires that loops involving top and charm quarks must be considered in the case of the kaon system. The mass difference Δm_K thus gets not only top box contributions but also charm-top and charm-charm contributions, which are long-distance contributions, difficult to estimate [24, 167]. A way out consists in considering observables related to CP violation in K^0 mixing and decays into pions. In the absence of CP violation, only the short-lived kaon K_S^0 decays into $\pi\pi$, whereas the long-lived kaon K_L^0 decays into 3π . A measurement of CP violation can be defined from the amplitude of K_S^0 and K_L^0 states decaying into a $\pi\pi$ state with total isospin $I = 0$

$$\epsilon_K = \frac{\langle (\pi\pi)_{I=0} | K_L^0 \rangle}{\langle (\pi\pi)_{I=0} | K_S^0 \rangle}. \quad (14)$$

This term is related to the difference between CP eigenstates and mass eigenstates and it requires a global fit to many observables describing $K \rightarrow 2\pi$ decays [25]. Its real part indicates CP violation in mixing and its imaginary part measures CP violation in the interference between mixing and decay. It turns out that ϵ_K can be computed accurately in terms of short-distance (Inami-Lim) functions as well as a long-distance bag parameter, which is known from lattice QCD simulations [26]. An accurate SM prediction of ϵ_K requires also a resummation of short-distance QCD corrections (gluon exchanges), encoded into $\eta_{tt}, \eta_{ct}, \eta_{cc}$. These coefficients have been computed up to NLO for η_{tt} [187] and NNLO for η_{ct}, η_{cc} [188, 189], where the latter is still affected by large theoretical uncertainties. The interpretation in terms of the CKM parameters involves A , $\bar{\rho}$ and $\bar{\eta}$ (and is thus connected with $|V_{cb}|$) and corresponds to a hyperbola in the $(\bar{\rho}, \bar{\eta})$ plane.

Another interesting quantity is given by ϵ'_K , defined to measure CP violation in decay by comparing rates of K_L^0 and K_S^0 into $\pi^+\pi^-$ and $\pi^0\pi^0$. It has been measured precisely [25, 190, 191], but it is difficult to predict theoretically, as it receives dominant contributions from two four-quark operators (denoted Q_6 and Q_8 in the framework of the effective Hamiltonian) which largely cancel each other. A lattice QCD evaluation of all the bag parameters needed has been performed recently [192], suggesting a discrepancy between 2 and 3 σ with respect to SM expectations [192–195]. This interesting but challenging issue definitely calls for estimations of the relevant bag parameters from other lattice QCD collaborations.

3.5 Lepton flavour universality

The metrology of the CKM parameters discussed up to now relies on modes that can be predicted accurately in the SM and provide information on its parameters. However it mixes modes with different sensitivities to physics beyond the SM: on one hand, flavour-changing charged currents, such as semileptonic decays, which are dominated by tree processes in the SM, and on the other hand, flavour-changing neutral currents, such as neutral-meson mixing, which are mediated by loop processes in the SM. Additional, rare, processes that are not expected to provide further constraints on the parameters of the SM can probe some of the underlying hypotheses at the core of this theory. More details can be found in a previous edition of this Review [196].

A particularly topical example is lepton-flavour universality. In both flavour-changing charged and neutral currents, the weak interaction at play deals with lepton flavours in a universal manner, whereas quarks are treated on a different footing due to the CKM matrix. This universality of lepton couplings is assumed when determining the CKM parameters, in particular to combine results from semileptonic and leptonic decays which involve e, μ and/or τ leptons.

Recently, LHCb and the B factories have provided interesting hints of violation of lepton-flavour universality in both flavour-changing charged and neutral currents [197]. The measurements in charged currents between $B \rightarrow D^{(*)}\tau\nu$ and $B \rightarrow D^{(*)}\ell\nu$ with $\ell = \mu, e$ [198–203] indicate that

the ratios $R(D)$ and $R(D^*)$ exceed SM predictions by 1.9σ and 3.3σ respectively, leading to a combined discrepancy with the SM at 4.0σ [42]

$$R_{D^{(*)}} = \frac{Br(B \rightarrow D^{(*)}\tau\nu)}{Br(B \rightarrow D^{(*)}\ell\bar{\nu}_\ell)}. \quad (15)$$

The individual branching ratios are consistent with a 15% enhancement for $b \rightarrow c\tau\bar{\nu}_\tau$ compared to SM expectations. Several similar measurements, notably from LHCb, are ongoing, which should give a clearer picture in the near future.

The violation of lepton flavour universality has also been discussed for the flavour-changing neutral current (FCNC) transition $b \rightarrow s\ell^+\ell^-$ at several experiments. The observable $R_K = Br(B \rightarrow K\mu^+\mu^-)/Br(B \rightarrow Ke^+e^-)$ was measured by LHCb [197] in the dilepton mass range from 1 to 6 GeV² as $0.745^{+0.090}_{-0.074} \pm 0.036$, corresponding to a 2.6σ tension with its SM value predicted to be equal to 1 (to a high accuracy). Other recent experimental results have shown interesting deviations from the SM in the muon sector. The LHCb analysis [204] of the decay $B^0 \rightarrow K^{*0}\mu^+\mu^-$ reports a $\sim 3\sigma$ anomaly in two large K^* -recoil bins of the angular observable P'_5 [205]. This was subsequently confirmed by the Belle experiment [206] with the hint that it would arise in $b \rightarrow s\mu^+\mu^-$ but not in $b \rightarrow se^+e^-$ [207, 208]. Finally, the LHCb results on the branching ratio of several $b \rightarrow s\mu^+\mu^-$ decays exhibit deviations at low dilepton masses [209–212].

If these deviations from lepton-flavour universality are confirmed it would be an unambiguous sign of physics beyond the SM. It would also have consequences on the constraints listed above, especially those in Sec. 3.2, which are determined using leptonic and semileptonic decays. Most analyses assume lepton universality, a hypothesis which would need to be revisited (see Sec. 4.2.2 for more detail).

4 Global analyses

4.1 Determination of CKM parameters

The following subsections describe how the above-mentioned individual constraints can be combined to constrain the CKM parameters.

4.1.1 Statistical approaches to global analyses

The individual constraints presented above must be combined in order to obtain statistically meaningful constraints on the CKM parameters. The problem can be described as a series of observables (e.g., branching ratios of leptonic and semileptonic decays, mass difference for neutral mesons...) depending on theoretical parameters, some of which are of interest ($A, \lambda, \bar{\rho}, \bar{\eta}$), the others being called “nuisance parameters” (e.g., decay constants, form factors, quark masses...). The statistical analysis aims especially at determining confidence intervals for the CKM parameters (and other fundamental parameters for models beyond the SM). The accuracy of the determination of the CKM parameters depends thus on the precision of the experimental measurements and the theoretical computations of nuisance parameters. Currently, global analyses are mainly limited by the latter, which are mostly obtained from QCD lattice simulations which consider a discretised version of QCD on a finite grid and compute correlators through Monte Carlo integrations over gluon gauge configurations. Due to the remarkable improvement in computing power and algorithms over the recent decades, these computations are now mainly dominated by systematic uncertainties (extrapolation in lattice spacing, volume and quark masses, renormalisation).

Therefore, a global analysis requires both a general statistical framework and a specific model for systematic uncertainties. Frequentist and Bayesian approaches have been proposed to deal

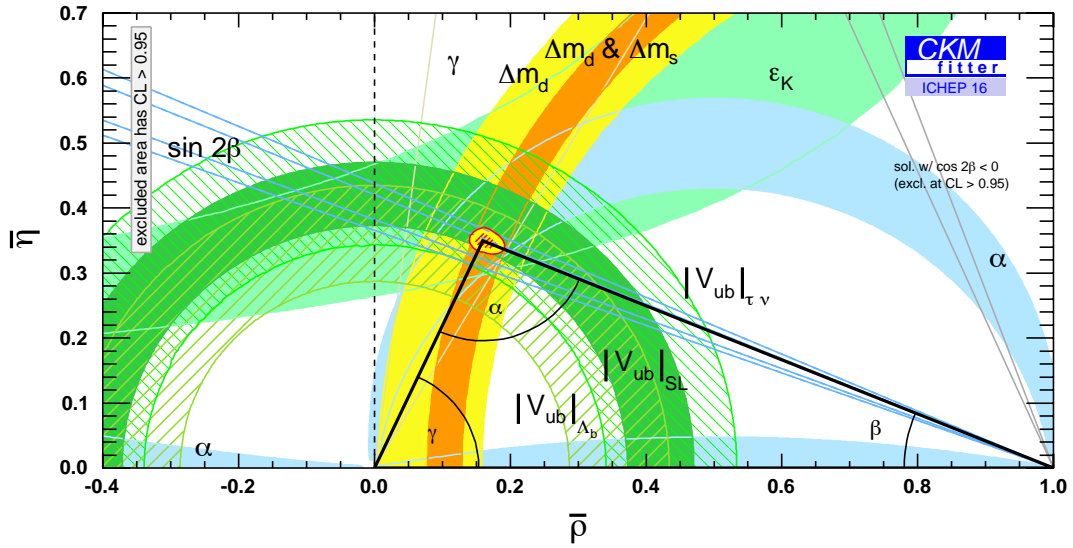


Figure 6: Status of the CKM unitarity triangle fit in $(\bar{\rho}, \bar{\eta})$ in Summer 2016. Regions outside the coloured areas have $\text{CL} > 95.45\%$. For the combined fit the yellow area inscribed by the contour line represents points with $\text{CL} < 95.45\%$. The shaded area inside this region represents points with $\text{CL} < 68.3\%$. Figure from Ref. [76].

with such analyses: the former defines probability as the outcome of repeated trials/measurements in the limit where their number becomes infinite, and the latter considers them as a subjective degree of credibility given by the observer to each possible result. The choice between the two approaches is a subject of considerable discussion in the literature (a specific discussion in the CKM case can be found in refs. [213–216]). The frequentist approach has been adopted by the CKMfitter group [155, 217], whereas a Bayesian approach is used by the UTfit group [218].

A second issue, the models for systematic uncertainties, is also a matter of debate. For lack of a better choice and even though they are not of statistical nature by definition, they are often described with the same model as statistical uncertainties, for instance in the case of the UTfit group [218]. Alternative treatments consist in determining sets of confidence intervals for specific values of the systematic uncertainties before combining them in unified confidence intervals (Scan method [219]) or building dedicated models for likelihoods and p-values treating a range of values for the systematic uncertainties on an equal footing (Rfit model used by the CKMfitter collaboration [155]). This choice has an impact not only when performing the global fit itself, but also when choosing inputs by averaging measurements or computations from different groups. A more detailed discussion of the various models for theoretical uncertainties can be found in Ref. [220].

4.1.2 Determination of the CKM parameters and consistency tests

For illustrative purposes, the results obtained by the CKMfitter group based on the results available at the time of the ICHEP 2016 conference [76] are used. The current situation of the global fit in the $(\bar{\rho}, \bar{\eta})$ plane is indicated in Fig. 6. The input parameters are listed in Table 2. As indicated in Sec. 2.2, this result could be cast into other unitary triangles.

Some comments are in order before discussing the metrology of the parameters. There exists a unique preferred region defined by the entire set of observables under consideration in the global fit. This region is represented by the yellow surface inscribed by the red contour line for which the values of $\bar{\rho}$ and $\bar{\eta}$ with a p -value such that $1 - p < 95.45\%$. The goodness of the fit

Table 2: Constraints used for the global fit, and the main inputs involved. When two uncertainties are quoted, the first one is statistical, the second one systematic. The lattice inputs and the averaging method used are discussed in Ref. [76], as well as additional theoretical inputs (quark masses, strong coupling constant, short-distance QCD corrections for meson mixing). For a review of lattice inputs see Ref. [26].

CKM	Process	Observables		Theoretical inputs	
$ V_{ud} $	$0^+ \rightarrow 0^+$ transitions	$ V_{ud} _{\text{nucl}}$	$= 0.97425 \pm 0 \pm 0.00022$	[41]	Nuclear matrix elements
$ V_{us} $	$K \rightarrow \pi \ell \nu$	$ V_{us} _{\text{SL}} f_+^{K \rightarrow \pi}(0)$	$= 0.2165 \pm 0.0004$	[25]	$f_+^{K \rightarrow \pi}(0) = 0.9681 \pm 0.0014 \pm 0.0022$
	$K \rightarrow e \nu_e$	$\mathcal{B}(K \rightarrow e \nu_e)$	$= (1.581 \pm 0.008) \cdot 10^{-5}$	[25]	$f_K = 155.2 \pm 0.2 \pm 0.6 \text{ MeV}$
	$K \rightarrow \mu \nu_\mu$	$\mathcal{B}(K \rightarrow \mu \nu_\mu)$	$= 0.6355 \pm 0.0011$	[25]	
	$\tau \rightarrow K \nu_\tau$	$\mathcal{B}(\tau \rightarrow K \nu_\tau)$	$= (0.6955 \pm 0.0096) \cdot 10^{-2}$	[25]	
$\frac{ V_{us} }{ V_{ud} }$	$K \rightarrow \mu \nu / \pi \rightarrow \mu \nu$	$\frac{\mathcal{B}(K \rightarrow \mu \nu_\mu)}{\mathcal{B}(\pi \rightarrow \mu \nu_\mu)}$	$= 1.3365 \pm 0.0032$	[25]	$f_K/f_\pi = 1.1959 \pm 0.0010 \pm 0.0029$
	$\tau \rightarrow K \nu / \tau \rightarrow \pi \nu$	$\frac{\mathcal{B}(\tau \rightarrow K \nu_\tau)}{\mathcal{B}(\tau \rightarrow \pi \nu_\tau)}$	$= (6.43 \pm 0.09) \cdot 10^{-2}$	[25]	
	νN	$ V_{cd} _{\nu N}$	$= 0.230 \pm 0.011$	[25]	
	$D \rightarrow \mu \nu$	$\mathcal{B}(D \rightarrow \mu \nu)$	$= (3.74 \pm 0.17) \cdot 10^{-4}$	[42]	$f_{D_s}/f_D = 1.175 \pm 0.001 \pm 0.004$
$ V_{cd} $	$D \rightarrow \pi \ell \nu$	$ V_{cd} f_+^{D \rightarrow \pi}(0)$	$= 0.1425 \pm 0.0019$	[36, 37]	$f_+^{D \rightarrow \pi}(0) = 0.666 \pm 0.020 \pm 0.048$
	$W \rightarrow c \bar{s}$	$ V_{cs} _{W \rightarrow c \bar{s}}$	$= 0.94_{-0.26}^{+0.32} \pm 0.13$	[25]	
	$D_s \rightarrow \tau \nu$	$\mathcal{B}(D_s \rightarrow \tau \nu)$	$= (5.55 \pm 0.24) \cdot 10^{-2}$	[42]	$f_{D_s} = 248.2 \pm 0.3 \pm 1.9 \text{ MeV}$
	$D_s \rightarrow \mu \nu$	$\mathcal{B}(D_s \rightarrow \mu \nu_\mu)$	$= (5.57 \pm 0.24) \cdot 10^{-3}$	[42]	
$ V_{ub} $	$D \rightarrow K \ell \nu$	$ V_{cs} f_+^{D \rightarrow K}(0)$	$= 0.7226 \pm 0.0034$	[36–38]	$f_+^{D \rightarrow K}(0) = 0.747 \pm 0.011 \pm 0.034$
	semileptonic B	$ V_{ub} _{\text{SL}}$	$= (3.98 \pm 0.08 \pm 0.22) \cdot 10^{-3}$	[42, 76]	form factors, shape functions
	$B \rightarrow \tau \nu$	$\mathcal{B}(B \rightarrow \tau \nu)$	$= (1.08 \pm 0.21) \cdot 10^{-4}$	[42]	$f_{B_s}/f_B = 1.205 \pm 0.003 \pm 0.006$
	semileptonic B	$ V_{cb} _{\text{SL}}$	$= (41.00 \pm 0.33 \pm 0.74) \cdot 10^{-3}$	[42]	form factors, OPE matrix elements
$ V_{ub}/V_{cb} $	semileptonic Λ_b	$\frac{\mathcal{B}(\Lambda_b \rightarrow p \mu^- \bar{\nu}_\mu)_{q^2 > 15}}{\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \mu^- \bar{\nu}_\mu)_{q^2 > 7}}$	$= (0.944 \pm 0.081) \times 10^{-2}$	[74]	$\frac{\zeta(\Lambda_b \rightarrow p \mu^- \bar{\nu}_\mu)_{q^2 > 15}}{\zeta(\Lambda_b \rightarrow \Lambda_c \mu^- \bar{\nu}_\mu)_{q^2 > 7}} = 1.471 \pm 0.096 \pm 0.290$
α	$B \rightarrow \pi \pi, \rho \pi, \rho \rho$	branching ratios, CP asymmetries		[42]	isospin symmetry
β	$B \rightarrow (c \bar{c}) K$	$\sin(2\beta)_{[c \bar{c}]}$	$= 0.691 \pm 0.017$	[42]	penguin neglected
γ	$B \rightarrow D^{(*)} K^{(*)}$	inputs for the 3 methods		[42]	GGSZ, GLW, ADS methods
ϕ_s	$B_s \rightarrow J/\psi(KK, \pi\pi)$	ϕ_s	$= -0.030 \pm 0.033$	[42]	penguin neglected
$V_{tq}^* V_{td}$	Δm_d	Δm_d	$= 0.5065 \pm 0.0019 \text{ ps}^{-1}$	[42]	$\hat{B}_{B_s}/\hat{B}_{B_d} = 1.007 \pm 0.014 \pm 0.014$
	Δm_s	Δm_s	$= 17.757 \pm 0.021 \text{ ps}^{-1}$	[42]	$\hat{B}_{B_s} = 1.320 \pm 0.016 \pm 0.030$
	$B_s \rightarrow \mu \mu$	$\mathcal{B}(B_s \rightarrow \mu \mu)$	$= (2.8_{-0.6}^{+0.7}) \cdot 10^{-9}$	[173]	$f_{B_s} = 225.1 \pm 1.5 \pm 2.0 \text{ MeV}$
	ϵ_K	$ \epsilon_K $	$= (2.228 \pm 0.011) \cdot 10^{-3}$	[25]	$\hat{B}_K = 0.7567 \pm 0.0021 \pm 0.0123$
$V_{td}^* V_{ts}$ $V_{cd}^* V_{cs}$					$\kappa_\epsilon = 0.940 \pm 0.013 \pm 0.023$

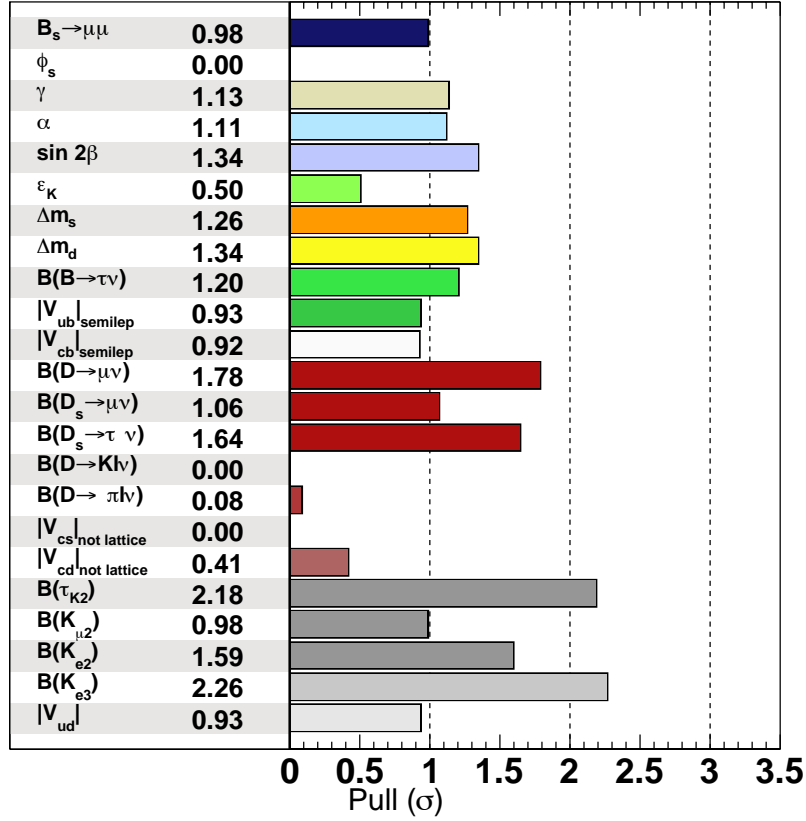


Figure 7: Pulls for the global fit in Summer 2016, as defined in Refs. [73, 76]. Each pull amounts to the absolute difference between predicted and measured value divided by the experimental uncertainty when the latter is large compared to the uncertainty of the prediction [221]. Figure from Ref. [76].

must be assessed in relation with the model used to describe theoretical uncertainties. If all the inputs uncertainties are assumed to be statistical in nature and can be combined in quadrature, the corresponding minimal χ^2 has a p -value of 20% (i.e., 1.3σ). The following values for the 4 parameters describing the CKM matrix are obtained

$$A = 0.825^{+0.007}_{-0.012}, \quad \lambda = 0.2251^{+0.0003}_{-0.0003}, \quad \bar{\rho} = 0.160^{+0.008}_{-0.007}, \quad \bar{\eta} = 0.350^{+0.006}_{-0.006}. \quad (16)$$

The overall consistency is striking when comparing constraints from tree- (leptonic and semileptonic decays) and loop-mediated (e.g., neutral-meson mixing) processes, as well as processes requiring CP violation (such as non-vanishing CP asymmetries) with respect to processes taking place even if CP were conserved (such as leptonic and semileptonic decays), as shown in Fig. 6. The consistency observed among the constraints allows one to perform the metrology of the CKM parameters and to give predictions for any CKM-related observable within the SM. Each comparison between the prediction issued from the fit and the corresponding measurement constitutes a null test of the SM hypothesis.

Some of the corresponding pulls are shown in Fig. 7, highlighting that there is no sign of discrepancy with this set of inputs. In particular, recent discrepancies related to $B(B \rightarrow \tau\nu)$, $\sin(2\beta)$, φ_s [73], V_{cb} , ϵ_K [222, 223] or $\Delta m_{d,s}$ [224] do not appear, either due to recent changes in the experimental inputs or to the dependence of these discrepancies on the statistical treatment and the modelling of systematic uncertainties.

Table 3: A few predictions from the global fit (indirect, i.e., not including direct determinations of these quantities) compared to the direct determinations: the first half corresponds to experimental inputs, the second half to inputs from lattice QCD computations. In the case of $Br(B_s^0 \rightarrow \mu^+ \mu^-)$, the value corresponds to the value before integration over time, i.e., removing the effect of $\Delta\Gamma_s$. Table from Ref. [76]. For $Br(B^0 \rightarrow \mu^+ \mu^-)$, an upper bound is available, but the statistical significance is too low to quote a measurement in the column “Direct determination”.

Quantity	Fit prediction	Direct determination
α ($^\circ$)	$92.1^{+1.5}_{-1.1}$	$88.8^{+2.3}_{-2.3}$
β ($^\circ$)	$23.7^{+1.1}_{-1.0}$	$21.8^{+0.7}_{-0.7}$
γ ($^\circ$)	$65.3^{+1.0}_{-2.5}$	$72.1^{+5.4}_{-5.8}$
φ_s (rad)	$-0.0370^{+0.0006}_{-0.0007}$	-0.030 ± 0.033
$Br(B_s^0 \rightarrow \mu^+ \mu^-) \times 10^9$	$3.36^{+0.07}_{-0.19}$	$2.62^{+0.66}_{-0.56}$
$Br(B^0 \rightarrow \mu^+ \mu^-) \times 10^{11}$	$9.55^{+0.25}_{-0.44}$	—
$V_{ub} \times 10^3$	$3.60^{+0.10}_{-0.10}$	$3.98 \pm 0.08 \pm 0.22$
$V_{cb} \times 10^3$	$42.2^{+0.7}_{-0.7}$	$41.00 \pm 0.33 \pm 0.74$
f_K	$0.15652^{+0.00013}_{-0.00020}$	$0.1552 \pm 0.0002 \pm 0.0006$
f_K/f_π	$1.1965^{+0.0021}_{-0.0063}$	$1.1959 \pm 0.0010 \pm 0.0029$
$f_+^{K \rightarrow \pi}(0)$	$0.9602^{+0.0020}_{-0.0025}$	$0.9681 \pm 0.0014 \pm 0.0022$
\hat{B}_K	$0.79^{+0.17}_{-0.11}$	$0.7567 \pm 0.0021 \pm 0.0123$
f_{D_s} (GeV)	$0.2512^{+0.0032}_{-0.0032}$	$0.2482 \pm 0.0003 \pm 0.0019$
f_{D_s}/f_D	$1.226^{+0.029}_{-0.027}$	$1.175 \pm 0.001 \pm 0.004$
$f_+^{D \rightarrow \pi}(0)$	$0.633^{+0.009}_{-0.008}$	$0.666 \pm 0.020 \pm 0.048$
$f_+^{D \rightarrow K}(0)$	$0.742^{+0.004}_{-0.004}$	$0.747 \pm 0.011 \pm 0.034$
$f_{B_s^0}$ (GeV)	$0.226^{+0.004}_{-0.005}$	$0.2251 \pm 0.0015 \pm 0.0020$
$f_{B_s^0}/f_B$	$1.243^{+0.027}_{-0.020}$	$1.205 \pm 0.003 \pm 0.006$
$\hat{B}_{B_s^0}$	$1.332^{+0.040}_{-0.067}$	$1.320 \pm 0.016 \pm 0.030$
$\hat{B}_{B_s^0}/\hat{B}_{B^0}$	$1.114^{+0.046}_{-0.047}$	$1.007 \pm 0.014 \pm 0.014$

Unitarity tests using direct determination of individual matrix elements (without resorting to unitarity) can also be performed by checking that the sum of their squares equals to unity. For the first two rows of the CKM matrix the following results are obtained

$$|V_{ud}|_{\text{meas}}^2 + |V_{us}|_{\text{meas}}^2 + |V_{ub}|_{\text{meas}}^2 - 1 = -0.0006^{+0.0006}_{-0.0002} \quad (17)$$

$$|V_{cd}|_{\text{meas}}^2 + |V_{cs}|_{\text{meas}}^2 + |V_{cb}|_{\text{meas}}^2 - 1 = -0.0034^{+0.0048}_{-0.0026} \quad (18)$$

where each “measured” value includes all semileptonic and leptonic direct determinations of a given CKM matrix element (an average of inclusive and exclusive semileptonic measurements is used for the semileptonic input for $|V_{ub}|$ and $|V_{cb}|$). No deviation from unitarity is observed. There is no direct determination of $|V_{td}|$ and $|V_{ts}|$ (they are obtained from $\Delta F = 2$ loop processes), as well as no accurate direct determination of $|V_{tb}|$ [25]: no equivalent test can thus be performed for the third row or any of the columns of the CKM matrix. Similarly, the value of $\alpha + \beta + \gamma$ cannot be probed directly, since the determination of α from $B \rightarrow \pi\pi, \pi\rho, \rho\rho$ relies already unitarity.

The global fit also provides indirect predictions (i.e., not including direct measurements of these quantities) for quantities of interest, either measured experimentally or determined from lattice QCD simulations, see Tab. 3. A similar level of accuracy is achieved for some observables in both their direct determinations and their indirect prediction. Improving their measurement

will have only a limited impact on the fit, unless the central value differs significantly from the global fit expectations (which would then require a fine understanding of all sources of uncertainties of the measurements). Other quantities are still far from being measured as accurately as their prediction from the global fit. Their measurements can help constraining further the CKM parameters and they still leave room for unexpected deviations from the SM picture emerging from the global fit.

4.2 Analyses of deviations from the CKM paradigm

Quark flavour physics provides both stringent tests of the SM and significant constraints on NP models. However, the above processes used to determine the CKM parameters show a good overall consistency within the SM, and thus lead to upper bounds on additional NP contributions. Additional processes suffering from larger theoretical or experimental uncertainties have thus to be included in the global analyses in order to probe physics beyond the SM.

Although specific NP models could be directly compared to experimental results, it proves natural for flavour processes to consider effective approaches. The short-distance dynamics is encoded in Wilson coefficients multiplied by operators describing the transition on long distances [24], since these flavour processes take place at significantly lower energies than the NP degrees of freedom of interest. New Physics affects the values of the Wilson coefficients. The structure of the operators affected (vector, scalar...) provides a hint of the type of NP at play, and the deviations of the Wilson coefficients from SM expectations provide an idea of the energy scales and coupling constants involved. In any case (specific NP models or general effective approaches) the above constraints must be reconsidered in order to determine whether they can be used to determine CKM parameters, constrain NP contributions, or neither.

Two topical examples are briefly discussed in the following subsections. The first is NP arising in $\Delta F = 2$ processes, and the second NP violating lepton flavour universality in $\Delta F = 1$ processes.

4.2.1 New Physics in $\Delta F = 2$

As discussed in Refs. [73, 177, 225–232] and in Sec. 3.4.1, neutral-meson mixing is a particularly interesting probe of NP. The evolution of the $B_q\bar{B}_q$ system is described through a quantum-mechanical Hamiltonian $H = M^q - i\Gamma^q/2$ as the sum of two Hermitian “mass” and “decay” matrices, so that $B_{(s)}^0\text{--}\bar{B}_{(s)}^0$ oscillations involve the off-diagonal elements M_{12}^q and Γ_{12}^q , respectively. The three physical quantities $|M_{12}^q|$, $|\Gamma_{12}^q|$ and $\varphi_q = \arg(-M_{12}^q/\Gamma_{12}^q)$ can be determined from the mass difference $\Delta m_q \simeq 2|M_{12}^q|$ among the eigenstates, their width difference $\Delta\Gamma_q \simeq 2|\Gamma_{12}^q|\cos\varphi_q$ and the semileptonic CP asymmetry $a_{\text{sl}}^q = \text{Im}\Gamma_{12}^q/M_{12}^q = \Delta\Gamma_q/\Delta m_q \tan\varphi_q$. Resulting from box diagrams with heavy (virtual) particles, M_{12}^q is expected to be especially sensitive to NP [73], so that the two complex parameters Δ_s and Δ_d , defined as

$$M_{12}^q \equiv M_{12}^{\text{SM},q} \Delta_q, \quad \Delta_q \equiv |\Delta_q| e^{i\varphi_q^\Delta}, \quad q = d, s, \quad (19)$$

can differ substantially from the SM value $\Delta_s = \Delta_d = 1$.

Importantly, the NP phases $\varphi_{d,s}^\Delta$ do not only affect $a_{\text{sl}}^{d,s}$, but also shift the CP phases extracted from the mixing-induced CP asymmetries in $B^0 \rightarrow J/\psi K_s^0$ and $B_s^0 \rightarrow J/\psi \phi$ to $2\beta + \varphi_d^\Delta$ and $2\beta_s - \varphi_s^\Delta$, respectively. If it is assumed that NP enters only through the two parameters Δ_d and Δ_s , the CKM paradigm is still valid to analyse $\Delta F = 1$ quark flavour transitions. On the contrary, the $\Delta F = 2$ transitions previously used to determine the CKM parameters must be reinterpreted as constraints on Δ_d and Δ_s (namely Δm_d , Δm_s , $\sin(2\beta)$ and α).

There has been a lot of interest for such NP scenarios triggered by deviations first observed in the early measurements from CDF and DØ on the B_s^0 mixing angle φ_s , and further once DØ

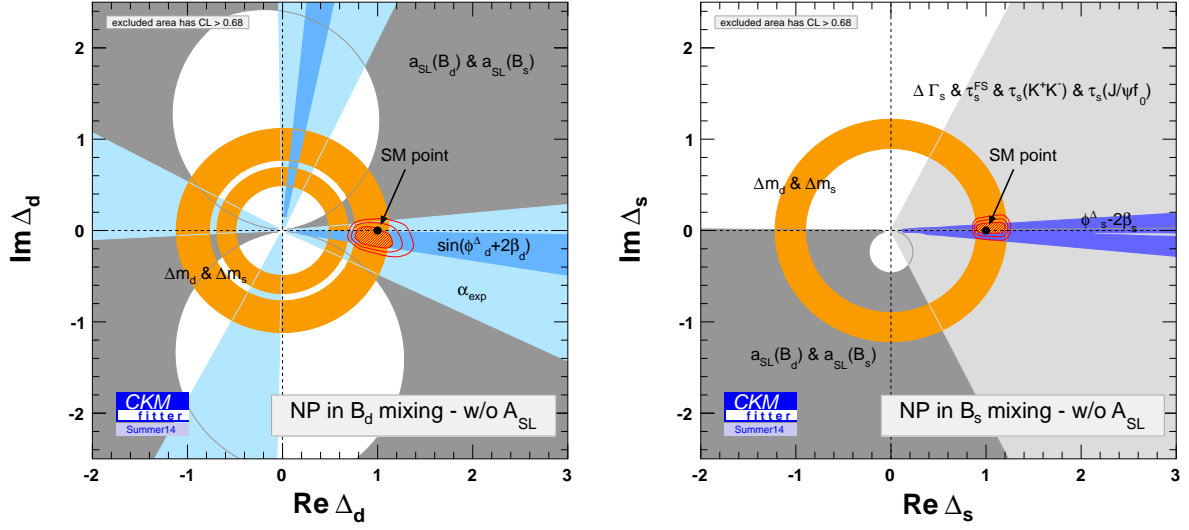


Figure 8: Complex parameters (left) Δ_d and (right) Δ_s describing NP in $\Delta F = 2$ (Scenario I, not including a_{sl}). The coloured areas represent regions with $1 - p < 68.3\%$ for the individual constraints. The red area shows the region with $1 - p < 68.3\%$ for the combined fit, with the two additional contours delimiting the regions with $1 - p < 95.45\%$ and $1 - p < 99.73\%$. Plot from Ref. [87].

quoted values of the like-sign dimuon asymmetry a_{sl} (measuring a linear combination of a_{sl}^d and a_{sl}^s). However, as discussed in Sec. 3.4.1, later measurements of the individual semileptonic CP asymmetries and mixing angles for B^0 and B_s^0 mesons have not been able to explain the $D\bar{O}$ measurement, as they showed a good agreement with SM expectations.

Simultaneous fits of the CKM parameters and the NP parameters Δ_d , and Δ_s have been performed [73, 177] in different generic scenarios in which NP is confined to $\Delta F = 2$ flavour-changing neutral currents. The most recent update [87] used data up to Summer 2014. The two complex NP parameters Δ_d and Δ_s are not sufficient to absorb the discrepancy between the $D\bar{O}$ measurement of a_{sl} and the rest of the global fit [87]. Without a_{sl} , the fit including NP in $\Delta F = 2$ is good, but the improvement with respect to the SM is limited. In the case of the so-called scenario I (Δ_s and Δ_d independent), the following values are obtained

$$\Delta_d = (0.94^{+0.18}_{-0.15}) + i(-0.12^{+0.12}_{-0.05}) \quad \Delta_s = (1.05^{+0.14}_{-0.13}) + i(0.03^{+0.04}_{-0.04}), \quad (20)$$

together with the values of the CKM parameters

$$A = 0.790^{+0.038}_{-0.008}, \quad \lambda = 0.2258^{+0.0005}_{-0.0006}, \quad \bar{\rho} = 0.136^{+0.022}_{-0.028}, \quad \bar{\eta} = 0.402^{+0.015}_{-0.054}. \quad (21)$$

The constraints are shown in Fig. 8. Data still allow sizeable NP contributions in both B^0 and B_s^0 sectors up to 30–40% at the 3σ level. The results for the CKM parameters can be compared to those of Eq. (16), keeping in mind that the inputs are different. Unsurprisingly, there is a larger range of variations of the CKM parameters once some of the constraints involve not only SM but also NP contributions.

The same kind of analysis has also been used for prospective studies taking into account the accuracies expected from the full data sets of the LHCb phase 1 upgrade and Belle II [232]. Assuming no signal of NP, the constraints on Δ_d and Δ_s tighten, setting stringent constraints on the scale of NP involved, which can range from 10 to 10^3 TeV, depending on the structure of couplings chosen.

4.2.2 Violation of lepton flavour universality in $\Delta F = 1$ processes

As discussed in Sec. 3.5, there are interesting hints of a breakdown of lepton flavour universality in both $b \rightarrow c\ell\nu$ and $b \rightarrow s\ell\ell$ processes. Both types of processes have been analysed to extract information on potential NP contributions in the effective Hamiltonian approach describing the process at the scale $\mu_b = O(m_b)$ around the b -quark mass after integrating out heavier degrees of freedom [24].

For the $b \rightarrow c\ell\nu$ transitions, the ratios of branching ratios $R(D)$ and $R(D^*)$ do not involve CKM parameters. The deviations can be easily interpreted by adding new interactions to the effective Hamiltonian, for instance additional NP scalar couplings [233]. A more extensive study [234] highlights a few scenarios compatible not only with the branching ratios, but also with the q^2 -shape of the $B \rightarrow D\tau\bar{\nu}_\tau$ differential decay rate. Two-dimensional scenarios with left- and right-handed couplings, either vector or scalar, are favoured. It should be noted that the $B \rightarrow D\ell\bar{\nu}_\ell$ form factors are known from lattice QCD simulations [235, 236], but this is not the case for the $B \rightarrow D^*\ell\bar{\nu}_\ell$ decay whose prediction requires many additional theoretical assumptions (validity of heavy quark effective theory, absence of NP for electrons and muons). Moreover, there are presently only a very limited number of observables (two ratios of branching ratios). The geometry of the decay products could add further information on the deviations observed in the branching ratios [237], as well as checking the q^2 -dependence of the differential decay rates for both vector and pseudo-scalar final mesons.

There is a much larger set of observables concerning $b \rightarrow s\ell^+\ell^-$ decays, with many different channels. The interest of a global analysis of such decays was clear much before the advent of B -factory and LHCb data [238]. The appearance of several tensions in different $b \rightarrow s\ell^+\ell^-$ channels is interesting since all these observables are sensitive to the same couplings $\mathcal{C}_{7,9,10}^{(\prime)}$ induced by the local four-fermion operators in the effective Hamiltonian approach

$$\begin{aligned}\mathcal{O}_9^{(\prime)} &= \frac{\alpha}{4\pi} [\bar{s}\gamma^\mu P_{L(R)} b] [\bar{\mu}\gamma_\mu \mu], & \mathcal{C}_9^{\text{SM}}(\mu_b) &= 4.07 \\ \mathcal{O}_{10}^{(\prime)} &= \frac{\alpha}{4\pi} [\bar{s}\gamma^\mu P_{L(R)} b] [\bar{\mu}\gamma_\mu \gamma_5 \mu], & \mathcal{C}_{10}^{\text{SM}}(\mu_b) &= -4.31 \\ \mathcal{O}_7^{(\prime)} &= \frac{\alpha}{4\pi} m_b [\bar{s}\sigma_{\mu\nu} P_{R(L)} b] F^{\mu\nu}, & \mathcal{C}_7^{\text{SM}}(\mu_b) &= -0.29\end{aligned}\tag{22}$$

where $P_{L,R}$ project on left- and right-handed chiralities and primed operators have vanishing or negligible $\mathcal{C}_{7,9,10}'$ Wilson coefficients in the SM. The couplings $\mathcal{C}_{7,9,10}^{(\prime)}$ can be constrained through various observables in radiative and (semi-) leptonic $B_{(s)}^0$ decays, each of them sensitive to different subsets and combinations of coefficients. The first analyses performed in this spirit and exploiting LHCb data [239] pointed to a large contribution to the Wilson coefficient \mathcal{C}_9 in $b \rightarrow s\mu^+\mu^-$, quickly confirmed by Refs. [240, 241]. Three recent global analyses [242–244] have been performed, involving similar sets of up-to-date data. They rely on different inputs and hypotheses but agree in their conclusions and prefer scenarios involving a significant contribution to $\mathcal{C}_9(m_b) \simeq -1.1$ in $b \rightarrow s\mu^+\mu^-$, whereas contribution to other Wilson coefficients are only loosely bound and compatible with the SM. An intense theoretical activity is currently going on to cross check the various sources of theoretical uncertainties (power corrections to the limit $m_b \rightarrow \infty$, form factors, long-distance charm-loop contributions [245–251]) confirming up to now the robustness of this picture.

As there is no clear picture for NP models that could be responsible for the deviations in both $b \rightarrow c\ell\nu$ and $b \rightarrow s\ell^+\ell^-$ decays (even though leptoquarks, Z' bosons, partial compositeness models are favoured), it is not easy to perform a combined fit of CKM parameters and NP contributions in a way similar to the $\Delta F = 2$ case reported in Sec. 4.2.1. Indeed, the NP analyses have often assumed values of the CKM parameters based either on full global fits or tree-level determinations, assuming that the uncertainty coming from CKM parameters is sub-leading

compared to other sources of uncertainties.

However, if there is a violation of lepton flavour universality, all leptonic and semileptonic decays may be significantly affected. Unfortunately, not all measurements are given for muonic and electronic modes separately. Removing all these modes from the determination of the CKM parameters leads to

$$\begin{aligned} A &= 0.831^{+0.058}_{-0.109}, \quad \lambda = 0.213^{+0.010}_{-0.005}, \quad \bar{\rho} = 0.127^{+0.019}_{-0.019}, \quad \bar{\eta} = 0.350^{+0.012}_{-0.011}, \\ |V_{cb}| &= 0.0421^{+0.0011}_{-0.0016}, \quad |V_{ts}| = 0.0414^{+0.0010}_{-0.0016}. \end{aligned} \quad (23)$$

A second approach is also possible, following the current experimental indications that electron modes are in agreement with SM. Only the μ and τ modes should then be removed from the global fit to the CKM parameters, leading to

$$\begin{aligned} A &= 0.831^{+0.021}_{-0.031}, \quad \lambda = 0.2251^{+0.0004}_{-0.0004}, \quad \bar{\rho} = 0.155^{+0.008}_{-0.008}, \quad \bar{\eta} = 0.340^{+0.010}_{-0.010}, \\ |V_{cb}| &= 0.0425^{+0.0007}_{-0.0018}, \quad |V_{ts}| = 0.0410^{+0.0014}_{-0.0012} \end{aligned} \quad (24)$$

In both cases, $|V_{tb}|$ is unity up to a very high accuracy. These results can be compared with those from the SM global fit Eq. (16):

$$|V_{cb}| = 0.0418^{+0.0003}_{-0.0006}, \quad |V_{ts}| = 0.0411^{+0.0003}_{-0.0006} \quad (25)$$

Removing part or all the modes potentially affected by the violation of lepton flavour universality increases significantly the uncertainties (up to a factor 5) on the CKM matrix elements $|V_{cb}|$ and $|V_{ts}|$ which arise in $b \rightarrow c\ell\nu$ and $b \rightarrow s\ell\ell$ decays, respectively. However, considering the other experimental and theoretical uncertainties involved, the parametric uncertainty coming from CKM parameters remains indeed sub-leading for the NP analyses of these modes and it should not alter their conclusions.

5 Outlook

The Cabibbo-Kobayashi-Maskawa matrix is a key element to describe flavour dynamics in the Standard Model. With only four parameters, this matrix is able to describe a large range of phenomena, such as CP violation and rare decays. It can thus be constrained by many different processes, which have to be measured experimentally with a high accuracy and computed with a good theoretical control. After the first LEP measurements, the turn of the millennium has opened the B -factory era leading to a remarkable improvement in the number and accuracy of the constraints set on the CKM matrix, exhibiting a remarkable consistency and leading to a precise determination of the CKM parameters.

The status presented in Sec. 3 is based on experiments up to and including the lifetime of the B factories, as well as the LHC Run 1. The corresponding data sets have been almost fully exploited, while no updated measurements using data from the ongoing Run 2 are yet available. This situation will soon change as first Run 2 analyses will be released by LHCb, ATLAS and CMS. A change of gear is expected after the year 2020 when both Belle II and the phase-1 upgraded LHCb experiment will collect data at much improved luminosities. The target is a multiplication of the data sets by up to two orders of magnitude [252, 253]. In the case of LHCb this includes the increase of the $b\bar{b}$ cross-section at higher energies and an improved trigger [254]. A reduction of experimental uncertainties by factors around ten on the angles β , γ and φ_s is foreseen as no irreducible systematic uncertainties are expected to affect the results in the foreseeable future. One may also expect improvements in the experimental measurements of the observables related to the angle α and the matrix elements $|V_{ub}|$ and $|V_{cb}|$. In addition new

measurements concerning lepton flavour universality and observables in rare decays are likely to be presented in the coming years.

The interpretation of these improved measurements will depend on developments in theoretical calculations. The computation using lattice QCD simulations has already reached a very mature stage for some of the quantities described in Sec. 4, for instance decay constants and form factors. At the accuracy obtained, some issues become relevant, such as the estimation of electromagnetic corrections, the detailed extrapolation in heavy quark masses, the kinematic range available for heavy-to-light form factors. Hopefully, the resulting improvement in the accuracy of the theoretical computations may resolve the puzzles currently affecting the determination of $|V_{ub}|$ and $|V_{cb}|$. More generally, the experimental accuracy reached for the individual constraints requires to reassess some of the theoretical hypotheses commonly used to extract these quantities and add systematic uncertainties which have been neglected up to now (for instance, sources of isospin breaking arising in the determination of α , penguin pollution for β ...). Other improvements can be expected concerning more exploratory domains, such as the matrix elements of operators beyond the Standard Model (needed to analyse flavour constraints in NP models) or quantities involving hadrons difficult to access up to now – for instance, unstable mesons decaying under the strong interaction (ρ , K^* ...), light or heavy baryons (nucleons, hyperons, Λ_b ...). Progress can also be expected from other theoretical methods (effective theories, dispersive approaches...). Even though it is more difficult to assess their impact on the study of the CKM matrix, these advances should help in the study of ϵ'/ϵ , the constraints on New Physics from neutral-meson mixing, or the interpretation of anomalies in rare b decays.

The current picture provided by global fits to CKM parameters within the SM is both accurate and consistent, and it shows that this approach can be used to study NP models affecting flavor dynamics (such as models with NP in $\Delta F = 2$ transitions). Such analyses extend the initial objective of constraining the CKM matrix, and they require a joint determination of the CKM parameters and NP contributions, based on a larger set of measured observables. This approach through global fits is currently relevant to study the hints of a violation of lepton flavour universality in $b \rightarrow c$ and $b \rightarrow s$ transitions which have sparked a lot of interest. Several attempts at analysing these deviations in terms of model-independent effective approaches exist, but it remains to connect these results with viable high-energy models. In these challenging analyses, the uncertainties related to CKM parameters in such analyses are sub-leading compared to other (experimental and hadronic) uncertainties. A consistent picture of whether lepton universality holds will hopefully soon become available and will provide original directions for these studies.

More generally, new developments in flavour physics can be expected through the improved determination of CKM parameters, the identification of departures from the Standard Model in flavour transitions and the study of heavy degrees of freedom through low-energy processes at high intensity. In all these aspects the upcoming measurements from LHCb and Belle-II and the ongoing progress in theoretical computations will play an essential role in the coming years.

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